This question paper contains 8 printed pages.]

6631 *Your Roll No. .....*

B.Sc. (H) / Computer Science / VI Sem. B

Paper-604

OPERATIONAL RESEARCH

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **all** questions.*

Use of calculator is permitted.

Marks are indicated against each question.

1. Two small scale production companies A and B plan to advertise their products on local radio or television. The advertisement budget is limited to ₹ 10,000 a month. Each minute of radio advertisement cost ₹ 15 and each minute of TV commercials costs ₹ 300. Both the companies would like to use radio advertisement at least twice as much as TV. In the mean time, it is not practical to use more than 400 minutes of radio advertisement a month. Past experience shows that TV advertisement is estimated 25 times more effective than

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that of radio, formulate the linear programming problem and solve it graphically for maximizing project. 8

2. Solve the following l.p.p.

Maximize $z = 5x_1 - 4x_2 + 3x_3$

Subject to

$2x_1 + x_2 - 6x_3 = 20$

$6x_1 + 5x_2 + 10x_3 \leq 76$

$8x_1 - 3x_2 + 6x_3 \leq 50$

$x_1, x_2, x_3 \geq 0$ 8

3. (a) Consider the following l.p.p.

Maximize $z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

Subject to

$x_1 + x_2 + x_3 = 4$

$x_1 + 4x_2 + x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0$

The optimal objective row is given as

$x + 2x_1 + 0x_2 + 0x_3 + 3x_4 = 16$

(i) Write the dual of above l.p.p.

(ii) Use above information to determine the associated optimal dual solution given that x_3 and x_4 are initial basic variables. 6

(b) How would you detect multiple optimal solutions and infeasible solution of a l.p.p. using simplex method. 3

4. (a) Solve the following transportation problem whose cost matrix availability at each plant and requirements at each warehouse are given as follows 8

Warehouses

Plant	W_1	W_2	W_3	W_4	Availability
P_1	15	24	11	12	5000
P_2	25	20	14	16	4000
P_3	12	12	22	13	7000
Requirement	3000	2500	3500	4000	

(b) Five salesmen are to be assigned to five territories. Based on the past preference, the following table shows the annual sales (in ₹ lakhs) that can be generated by each salesman in each territory. Find the optimum assignment to maximize the sales. 6

	Territory				
Salesman	T_1	T_2	T_3	T_4	T_5
S_1	26	14	10	12	9
S_2	31	27	30	14	16
S_3	15	18	16	25	30
S_4	17	12	21	30	25
S_5	20	19	25	16	10

5. A truck can carry a total of 5 tons of product. Three types of product are available for shipment. Their weights and values are given in the table below :

i = Type of items	Weight (tons) w_i	Value (Rs.) h_i
A	1	20
B	2	50
C	2	60

(a) Write the mathematical model of the knapsack problem to determine the most valuable cargo without exceeding the maximum cargo weight for the above data. 3

(b) Use dynamic programming technique to solve the mathematical model. 7

6. (a) Solve the 4 × 2 game for the player B whose pay off matrix is as follows:

Player B

$$\text{Player A} \quad \begin{matrix} & B_1 & B_2 \\ A_1 & 6 & 5 \\ A_2 & 3 & 6 \\ A_3 & 8 & 4 \\ A_4 & 7 & -1 \end{matrix}$$ 6

(b) Consider the following Quadratic Programming Problem

$$\text{Max. } Z = C^T X + \frac{1}{2} X^T Q X$$

$$\text{S. To } AX \leq b$$

$$X \geq 0.$$

Where Q is a n×n symmetric matrix, A, C and b have appropriate dimensions. Form the corresponding Lograngian function. Write the KKT conditions to get an optimal solution. Write the corresponding auxillary linear programming problem to be solved by Wolfe's method. 5

7. (a) What is meant by queue discipline ? Explain various types of queue disciplines. 3

(b) A road transport company has one reservation clerk on duty at a time. He handles information of bus schedules and makes reservations. Customers

arrive at a rate of 8 per hour and the clerk can service 12 customers on an average per hour. Identify the appropriate queuing model. Further, find

(i) the average number of customers waiting for the service of the clerk. 1

(ii) the average time a customer has to wait before getting serviced. 1

(iii) probability that the queue size exceeds 5. 2

8. A small project is composed of 6 activities whose time estimates are listed in the table below :

	Duration (days)		
Activity	Optimistic	Most likely	Pessimistic
1-2	1	4	7
1-4	4	6	14
2-3	3	3	3
2-4	4	10	22
2-5	5	7	15
3-4	0	0	0
4-6	2	5	14
5-6	4	4	4

(a) Draw the project network and identify the critical path.

(b) Determine the mean project completion time.

(c) Find the probability that the project is completed in 18 days.

(d) What project duration will have 99% confidence of completion ? 8

Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	1591	.1628	.1664	1700	1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Also, for z = 4.0, 5.0, and 6.0, the probabilities are 0.49997, 0.4999997, and 0.499999999.